Solving Logarithms and Exponentials

Name____________________________

1) **Solve $2^x = 30$ by two different methods describing your strategy.**

Take log of both sides $\log 2^x = \log 30$

Power rule (#3) $x \cdot \log 2 = \log 30$

Divide $x = \frac{\log 30}{\log 2}$

$\boxed{x = 4.9069}$

$\log_2 30 = x$ Switch to log form

$\frac{\log 30}{\log 2} = x$ Use change of base formula

$\boxed{4.9069 = x}$

2) **Solve $10^{2x} = 52$**

$\log_{10} 10^{2x} = \log_{10} 52$

$2x \cdot \underset{1}{\log_{10} 10} = \log 52$

$2x = \log 52$

$x = \frac{\log 52}{2}$

$\boxed{x = .8580}$

4) **$2e^x + 5 = 115$**

$\quad -5 \quad -5$

$\frac{2e^x}{2} = \frac{110}{2}$

$\ln e^x = \ln 55$

$x = \ln 55$

$\boxed{x = 4.0073}$

3) **Solve $\frac{3(2^{x+4})}{3} = \frac{350}{3}$**

$\log 2^{(x+4)} = \log \frac{350}{3}$

$\frac{(x+4)(\log 2)}{\log 2} = \frac{\log\left(\frac{350}{3}\right)}{\log 2}$

$x + \underset{-4}{4} = \underset{-4}{6.8662}$

$\boxed{x = 2.8662}$

5) $\underset{-7}{7} + 15e^{1-3z} = \underset{-7}{10}$

$\frac{15 \cdot e^{(1-3z)}}{15} = \frac{3}{15}$

$\cancel{\ln} e^{(1-3z)} = \ln \frac{1}{5}$

$\underset{-1}{1} - 3z = \ln\left(\frac{1}{5}\right) - 1$

$\frac{-3z}{-3} = \frac{-2.6094}{-3}$

$\boxed{z = .8698}$

6) **Solve $x - xe^{5x+2} = 0$**

More than one x, factor $x(1 - e^{(5x+2)}) = 0$

zero product property

$x = 0$ $\quad \underset{-1}{1} - e^{5x+2} = \underset{-1}{0}$

$\frac{-e^{5x+2}}{-1} = \frac{-1}{-1}$

$\ln e^{5x+2} = \ln 1$

$5x + 2 = \ln 1$

$\frac{5x}{5} = \frac{\ln 1 - 2}{5}$

$\boxed{x = -.4}$

7) Solve $5(x^2 - 4) = (x^2 - 4)e^{7-x}$

$5(x^2-4) - (x^2-4)e^{(7-x)} = 0$ Set = to zero

$(x^2-4)(5 - e^{(7-x)}) = 0$ GCF

$x - 2 = 0 \quad x + 2 = 0 \quad \underset{-5}{5} - e^{(7-x)} = \underset{-5}{0}$ ZPP

$\boxed{x = 2} \quad \boxed{x = -2}$

$-e^{(7-x)} = -5$

$\ln e^{(7-x)} = \ln 5$

$(\underset{-7}{7}-x)\cancel{\ln e} = \underset{-7}{\ln 5}$

$-x = \ln 5 - 7$ $\quad \boxed{x = 5.3906}$

8) There were 20 rabbits on an island. After six months the number of rabbits had increased to 100.

If the number of rabbits increased exponentially, then how many rabbits will there be at the end of one year?

$A = Pe^{rt}$ $\quad$ t-time in years

$\frac{100}{20} = \frac{20e^{r(.5)}}{20}$

$\ln 5 = \cancel{\ln} e^{.5r}$

$\frac{\ln 5}{.5} = \frac{.5r \cdot (1)}{.5}$

$\boxed{3.2189 = r}$

so after 1 year

$A = 20e^{3.2189(1)}$

$\boxed{A = 500 \text{ rabbits}}$ I used all decimals

9) 3 years ago you had \$120 in the bank. You now have \$192.

If the amount of money at the end of each year increases exponentially, then how much will you have in the bank after another 7 years? Compounded annually

(Assume that you do not deposit or withdraw any money)

$y = ab^x$

$\frac{192}{120} = \frac{120(b)^3}{120}$

Cube root $\frac{192}{120} = b^3$

$\sqrt[3]{\frac{192}{120}} = b$

$1.1696 = b$

After 7 years

$y = 120(1.1696)^7$

$y = \$359.30$

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10) In mediaeval times there were 10,000 people living in a city that was struck by a plague so that people began to die at an exponential rate daily.

After 6 days, there were only 8,500 people living. How many were living after three weeks?

$A = Pe^{rt}$ $\quad$ t-days

$8500 = 10000e^{r(6)}$

$\ln \frac{85}{100} = \ln e^{6r}$

$\ln\left(\frac{85}{100}\right) = 6r$ $\quad \boxed{-.0271 = r}$

$A = 10000e^{-.0271(21)}$

$\boxed{A = 5660.35}$

people remain

11) A scientist started with a culture of 20 bacteria in a dish.

The number of bacteria at the end of each successive hour increased exponentially, so that the number at the end of one day was 220.

To the nearest million, how many bacteria were there after one week?

$A = Pe^{rt}$ $\quad$ t-hours $\quad$ or $\quad$ t-days

$220 = 20e^{r(24)}$

$11 = e^{r24}$

$\ln 11 = 24r$

$.0999 = r$

$A = 20e^{.0999(168)}$

$\boxed{A = 389743418}$

difference in rounding in work

$\frac{220}{20} = \frac{20e^{r(1)}}{20}$

$\ln 11 = \ln e^r$

$\ln 11 = r$

$2.3979 = r$

$A = 20e^{2.3979(7)}$

$\boxed{A = 389743420}$

bacteria

12) The number of people living in a country is increasing each year exponentially so that the number of people 5 years ago was 4 million.

The number of people in five years time is projected to be 6.25 million.

10 year time span

What is the present population of the country?

$A = Pe^{rt}$

$6.25 = 4e^{r(10)}$

$\ln \frac{6.25}{4} = \cancel{\ln} e^{10r}$

$\frac{\ln\left(\frac{6.25}{4}\right)}{10} = \frac{10r}{10}$

$.0446 = r$

$A = 4e^{.0446(5)}$

$\boxed{\text{A = 5 million people at present}}$

13) An \$1,000 investment is made in a trust fund at an annual percentage rate of 12%, compounded monthly. How long will it take the investment to reach \$2,000?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$2000 = 1000\left(1 + \frac{.12}{12}\right)^{12t}$

\$ doubles super common question → $2 = (1.01)^{12t}$

Take log of both sides

$\log 2 = 12t \cdot \log(1.01)$

$\frac{\log 2}{12 \log(1.01)} = t$

$\boxed{5.8 = t}$ years

14) A \$5,000 investment is made in a trust fund at an annual percentage rate of 10%, compounded annually. How long will it take the investment to reach \$15,500? Suppose that another bank promised you that your account would reach \$15,500 in 10 years, what annual interest would the second bank be paying?

Bank 1 $15{,}500 = 5000(1.10)^t$

$\log 3.1 = \log(1.10)^t$

$\log 3.1 = t * \log(1.10)$

$\frac{\log 3.1}{\log 1.10} = t$

$\boxed{11.8707 = t}$ years

Bank 2

$15{,}500 = 5000(r)^{10}$

$3.1 = r^{10}$

10th root

$1.11979 = r$

$\boxed{\text{12\% interest rate}}$

15) Show two different ways to solve $\$1,750 = \$1,000(1 + r)^5$

$\frac{1750}{1000} = \frac{1{,}000(1+r)^5}{1000}$

$1.75 = (1+r)^5$

Take the log of both sides

$\log 1.75 = \log(1+r)^5$

$\frac{\log 1.75}{5} = \frac{5}{5} \cdot \log(1+r)$

Write in exp form. $.0486076 = \log_{10}(1+r)$

$\underset{-1}{10^{.0486076}} = \underset{-1}{1 + r}$

$.1184269 = r$ $\quad \boxed{\text{rate 11.84\%}}$

$\frac{1750}{1000} = \frac{1{,}000(1+r)^5}{1000}$

$1.75 = (1+r)^5$ $\quad$ 5th root (Math button on calc)

$\underset{-1}{1.1184269} = \underset{-1}{1 + r}$

$.1184 = r$

$\boxed{\text{11.84\% rate}}$