Solving Logarithms and Exponentials Name_____

1) **Solve $2^x = 30$ by two different methods describing your strategy.**

2) **Solve $10^{2x} = 52$**

3) **Solve $3(2^{x+4}) = 350$**

4) **$2e^x + 5 = 115$**

5) $7 + 15e^{1-3z} = 10$

6) Solve $x - xe^{5x+2} = 0$

7) Solve $5\left(x^2-4\right)=\left(x^2-4\right)\mathrm{e}^{7-x}$

8) There were 20 rabbits on an island. After six months the number of rabbits had increased to 100.
If the number of rabbits increased exponentially, then how many rabbits will there be at the end of one year?

9) 3 years ago you had $120 in the bank. You now have $192.
If the amount of money at the end of each year increases exponentially, then how much will you have in the bank after another 7 years?
(Assume that you do not deposit or withdraw any money)

10) In mediaeval times there were 10,000 people living in a city that was struck by a plague so that people began to die at an exponential rate daily.

After 6 days, there were only 8,500 people living. How many were living after three weeks?

11) A scientist started with a culture of 20 bacteria in a dish.
The number of bacteria at the end of each successive hour increased exponentially, so that the number at the end of one day was 220.

To the nearest million, how many bacteria were there after one week?

12) The number of people living in a country is increasing each year exponentially so that the number of people 5 years ago was 4 million.
The number of people in five years time is projected to be 6.25 million.

What is the present population of the country?

13) An $1,000 investment is made in a trust fund at an annual percentage rate of 12%, compounded monthly. How long will it take the investment to reach $2,000?

14) A $5,000 investment is made in a trust fund at an annual percentage rate of 10%, compounded annually. How long will it take the investment to reach $15,500? Suppose that another bank promised you that your account would reach $15,500 in 10 years, what annual interest would the second bank be paying?

Bank 1

Bank 2

15) Show two different ways to solve $\$1,750 = \$1,000\,(1+r)^5$

APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EARTHQUAKE WORD PROBLEMS: As with any word problem, the trick is convert a narrative statement or question to a mathematical statement.

Before we start, let's talk about earthquakes and how we measure their intensity.

In 1935 Charles Richter defined the magnitude of an earthquake to be

$$M = \log \frac{I}{S}$$

where I is the intensity of the earthquake (measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake) and S is the intensity of a "standard earthquake" (whose amplitude is 1 micron $=10^{-4}$ cm).

The **magnitude, M,** of a standard earthquake is

$$M = \log \frac{S}{S} = \log 1 = 0$$

Richter studied many earthquakes that occurred between 1900 and 1950. The largest had magnitude of 8.9 on the Richter scale, and the smallest had magnitude 0. This corresponds to a ratio of intensities of 800,000,000, so the Richter scale provides more manageable numbers to work with.

Each number increase on the Richter scale indicates an intensity ten times stronger. For example, an earthquake of magnitude 6 is ten times stronger than an earthquake of magnitude 5. An earthquake of magnitude 7 is $10 \times 10 = 100$ times strong than an earthquake of magnitude 5. An earthquake of magnitude 8 is $10 \times 10 \times 10 = 1000$ times stronger than an earthquake of magnitude 5.

Example 1: Early in the century the earthquake in San Francisco registered 8.3 on the Richter scale. In the same year, another earthquake was recorded in South America that was four time stronger. What was the magnitude of the earthquake in South American?

Subscripts are used to identify which location is being looked at.

Solution: Convert the first sentence to an equivalent mathematical sentence or equation.

$$\begin{aligned} M_{SF} &= \log \frac{I_{SF}}{S} = 8.3 \\ 8.3 &= \log \frac{I_{SF}}{S} \end{aligned}$$

Convert the second sentence to an equivalent mathematical sentence or equation.

$$\begin{aligned} M_{SA} &= \log \frac{I_{SA}}{S} \\ I_{SA} &= 4I_{SF} \\ M_{SA} &= \log \frac{4I_{SF}}{S} \end{aligned}$$

Solve for M_{SA}.

$$\begin{aligned} M_{SA} &= \log \frac{4I_{SF}}{S} \\ &= \log 4I_{SF} - \log S \\ &= \log 4 + \log I_{SF} - \log S \\ &= \log 4 + (\log I_{SF} - \log S) \\ &= \log 4 + \log \frac{I_{SF}}{S} \\ &= \log 4 + 8.3 \\ &= 0.602059991328 + 8.3 \\ &= 8.90205999133 \\ M_{SA} &= 8.9 \end{aligned}$$

The intensity of the earthquake in South America was 8.9 on the Richter scale.

Example 2: A recent earthquake in San Francisco measured 7.1 on the Richter scale. How many times more intense was the San Francisco earthquake described in Example 1?

Solution: The intensity (I) of each earthquake was different. Let I_1 represent the intensity the early earthquake and I_2 represent the latest earthquake.

$$First \quad : \quad 8.3 = \log \frac{I_1}{S}$$

$$Second \quad : \quad 7.1 = \log \frac{I_2}{S}$$

What you are looking for is the ratio of the intensities: $\frac{I_1}{I_2}$. So our task is to isolate this ratio from the above given information using the rules of logarithms.

$$\begin{aligned}
\log \frac{I_1}{S} - \log \frac{I_2}{S} &= 8.3 - 7.1 \\
\left(\log \frac{I_1}{S}\right) - \left(\log \frac{I_2}{S}\right) &= 8.3 - 7.1 \\
(\log I_1 - \log S) - (\log I_2 - \log S) &= 1.2 \\
\log I_1 - \log S - \log I_2 + \log S &= 1.2 \\
\log I_1 - \log I_2 &= 1.2 \\
\log \frac{I_1}{I_2} &= 1.2
\end{aligned}$$

Convert the logarithmic equation to an exponential equation.

$$\begin{aligned}
\log \frac{I_1}{I_2} &= 1.2 \\
10^{1.2} &= \frac{I_1}{I_2} \\
\frac{I_1}{I_2} &= 15.8489319246 \\
\frac{I_1}{I_2} &\approx 16
\end{aligned}$$

The early earthquake was 16 times as intense as the later earthquake.