HPC B Log Rules

LAWS OF LOGARITHMS

Let a be a positive number, with $a \neq 1$. Let $A > 0$, $B > 0$, and C be any real numbers.

Law	Description
1. $\log_a(AB) = \log_a A + \log_a B$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$	The logarithm of a quotient of numbers is the difference of the logarithms of the numbers.
3. $\log_a(A^C) = C\log_a A$	The logarithm of a power of a number is the exponent times the logarithm of the number.

READ formulas and examples CAREFULLY! These are the rules that we covered in class. Remember that they are related to the exponential rules you already know. The notation at the beginning is only to state the "captain obvious" rules that it would be silly to discuss log base 1 and that we cannot take the log of a negative number.

Please recall from Algebra 2B switching from logarithm form to exponential form.

$\text{Log}_b\, x = y$ is the same as $b^y = x$ meaning $5^3 = 125$ is the same as $\text{Log}_5\, 125 = 3$

"Logs solve for exponents" How many times have you heard me say that ☺

Try and check a few:

1–6 ■ Express the equation in exponential form.

1. (a) $\log_2 32 = 5$ (b) $\log_5 1 = 0$
2. (a) $\log_{10} 0.1 = -1$ (b) $\log_8 512 = 3$
3. (a) $\log_4 2 = \frac{1}{2}$ (b) $\log_2\left(\frac{1}{16}\right) = -4$

7–12 ■ Express the equation in logarithmic form.

7. (a) $2^3 = 8$ (b) $10^{-3} = 0.001$
8. (a) $10^4 = 10{,}000$ (b) $81^{1/2} = 9$
9. (a) $4^{-3/2} = 0.125$ (b) $7^3 = 343$

EXAMPLE 1 ■ Using the Laws of Logarithms to Expand Expressions

Use the Laws of Logarithms to rewrite each expression.

(a) $\log_2(6x)$ (b) $\log\sqrt{5}$

(c) $\log_5(x^3y^6)$ (d) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

SOLUTION

(a) $\log_2(6x) = \log_2 6 + \log_2 x$ Law 1

(b) $\log\sqrt{5} = \log 5^{1/2} = \frac{1}{2}\log 5$ Law 3

(c) $\log_5(x^3y^6) = \log_5 x^3 + \log_5 y^6$ Law 1

$= 3\log_5 x + 6\log_5 y$ Law 3

(d) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right) = \ln(ab) - \ln\sqrt[3]{c}$ Law 2

$= \ln a + \ln b - \ln c^{1/3}$ Law 1

$= \ln a + \ln b - \frac{1}{3}\ln c$ Law 3

EXAMPLE 2 ■ Using the Laws of Logarithms to Evaluate Expressions

Evaluate each expression.

(a) $\log_4 2 + \log_4 32$ (b) $\log_2 80 - \log_2 5$ (c) $-\frac{1}{3}\log 8$

SOLUTION

(a) $\log_4 2 + \log_4 32 = \log_4(2 \cdot 32)$ Law 1

$= \log_4 64 = 3$ Because $4^3 = 64$

(b) $\log_2 80 - \log_2 5 = \log_2\left(\frac{80}{5}\right)$ Law 2

$= \log_2 16 = 4$ Because $2^4 = 16$

(c) $-\frac{1}{3}\log 8 = \log 8^{-1/3}$ Law 3

$= \log\left(\frac{1}{2}\right)$ Property of negative exponents

≈ -0.301 Use a calculator

4.4 EXERCISES

1–24 ■ Use the Laws of Logarithms to rewrite the expression in a form with no logarithm of a product, quotient, or power.

1. $\log_2(x(x-1))$ **2.** $\log_5\left(\frac{x}{2}\right)$

3. $\log 7^{23}$ **4.** $\ln(\pi x)$

5. $\log_2(AB^2)$ **6.** $\log_6\sqrt[4]{17}$

7. $\log_3(x\sqrt{y})$ **8.** $\log_2(xy)^{10}$

9. $\log_5\sqrt[3]{x^2+1}$ **10.** $\log_a\left(\frac{x^2}{yz^3}\right)$

11. $\ln\sqrt{ab}$ **12.** $\ln\sqrt[3]{3r^2s}$

13. $\log\left(\frac{x^3y^4}{z^6}\right)$ **14.** $\log\left(\frac{a^2}{b^4\sqrt{c}}\right)$

15. $\log_2\left(\frac{x(x^2+1)}{\sqrt{x^2-1}}\right)$ **16.** $\log_5\sqrt{\frac{x-1}{x+1}}$

17. $\ln\left(x\sqrt{\frac{y}{z}}\right)$ **18.** $\ln\frac{3x^2}{(x+1)^{10}}$

19. $\log\sqrt[4]{x^2+y^2}$ **20.** $\log\left(\frac{x}{\sqrt[3]{1-x}}\right)$

21. $\log\sqrt{\frac{x^2+4}{(x^2+1)(x^3-7)^2}}$

22. $\log\sqrt{x\sqrt{y\sqrt{z}}}$

23. $\ln\left(\frac{z^4\sqrt{x}}{\sqrt[3]{y^2+6y+17}}\right)$ **24.** $\log\left(\frac{10^x}{x(x^2+1)(x^4+2)}\right)$

Try and check a few:

Answers to the odds are at the end

35–44 ■ Rewrite the expression as a single logarithm.

35. $\log_3 5 + 5\log_3 2$

36. $\log 12 + \frac{1}{2}\log 7 - \log 2$

37. $\log_2 A + \log_2 B - 2\log_2 C$

38. $\log_5(x^2-1) - \log_5(x-1)$

39. $4\log x - \frac{1}{3}\log(x^2+1) + 2\log(x-1)$

40. $\ln(a+b) + \ln(a-b) - 2\ln c$

41. $\ln 5 + 2\ln x + 3\ln(x^2+5)$

42. $2(\log_5 x + 2\log_5 y - 3\log_5 z)$

43. $\frac{1}{3}\log(2x+1) + \frac{1}{2}[\log(x-4) - \log(x^4-x^2-1)]$

44. $\log_a b + c\log_a d - r\log_a s$

Remember $\ln$ is the same as $\log_e$ but we write $\ln$ because it is a special logarithm that is frequently used.

We write this in exponential form and take the logarithm, with base a, of each side.

$$b^y = x \quad \text{Exponential form}$$

$$\log_a(b^y) = \log_a x \quad \text{Take } \log_a \text{ of each side}$$

$$y\log_a b = \log_a x \quad \text{Law 3}$$

$$y = \frac{\log_a x}{\log_a b} \quad \text{Divide by } \log_a b$$

This proves the following formula.

CHANGE OF BASE FORMULA

$$\log_b x = \frac{\log_a x}{\log_a b}$$

We can now evaluate a logarithm to *any* base by using the Change of Base Formula to express the logarithm in terms of common logarithms or natural logarithms and then using a calculator.

45–52 ■ Use the Change of Base Formula and a calculator to evaluate the logarithm, correct to six decimal places. Use either natural or common logarithms.

45. $\log_2 7$ **46.** $\log_5 2$ **47.** $\log_3 11$

48. $\log_6 92$ **49.** $\log_7 3.58$ **50.** $\log_6 532$

EXAMPLE 5 ■ **Using the Change of Base Formula to Evaluate Logarithms**

Use the Change of Base Formula and common or natural logarithms to evaluate each logarithm, correct to five decimal places.

(a) $\log_8 5$ (b) $\log_9 20$

SOLUTION

(a) We use the Change of Base Formula with $b = 8$ and $a = 10$ to convert to common logarithms:

$$\log_8 5 = \frac{\log_{10} 5}{\log_{10} 8} \approx 0.77398$$

(b) We use the Change of Base Formula with $b = 9$ and $a = e$ to convert to natural logarithms:

$$\log_9 20 = \frac{\ln 20}{\ln 9} \approx 1.36342$$

Try and check a few:

25–34 ■ Evaluate the expression.

25. $\log_5\sqrt{125}$ **26.** $\log_2 112 - \log_2 7$

27. $\log 2 + \log 5$ **28.** $\log\sqrt{0.1}$

29. $\log_4 192 - \log_4 3$ **30.** $\log_{12} 9 + \log_{12} 16$

Answers to rewriting forms:

1-6

1(a) $2^5=32$ (b) $5^0=1$

2(a) $10^{-1}=.1$ (b) $8^3=512$

3(a) $4^{1/2}=2$ (b) $2^{-4}=\frac{1}{16}$

7-12

7(a) $\log_2 8=3$ (b) $\log 0.001=-3$

8(a) $\log 10000=4$ (b) $\log_{81} 9=\frac{1}{2}$

9(a) $\log_4 0.125=-\frac{3}{2}$ (b) $\log_7 343=3$

Answers to evens:

2. $\log_5 x - \log_5 2$
4. $\ln \pi + \ln x$
6. $\log_6 (17)^{1/2}$

   $\frac{1}{2}\log_6 17$
8. $10(\log_2 x + \log_2 y)$

   or

   $10\log_2 x + 10\log_2 y$
10. $\log_a x^2 - \log_a y - \log_a z^3$

    which is

    $2\log_a x - \log_a y - 3\log_a z$
12. $\frac{1}{3}(\ln 3 + 2\ln r + \ln s)$
14. $2\log a - 4\log b - \frac{1}{2}\log c$
16. $\frac{1}{2}(\log_5(x-1) - \log_5(x+1))$

    or

    $\frac{1}{2}\log_5(x-1) - \frac{1}{2}\log_5(x+1)$
18. $\ln 3 + 2\ln x - 10\ln(x+1)$
20. $\log x - \frac{1}{3}\log(1-x)$
22. $\frac{1}{2}\log x + \frac{1}{4}\log y + \frac{1}{8}\log z$
24. $x\log 10 - \log x - \log(x^2+1) - \log(x^4+2)$

36. $\log\left(\frac{12\sqrt{7}}{2}\right)$ or $\log 6\sqrt{7}$
38. $\log_5\left(\frac{x^2-1}{x-1}\right)$ or $\log_5(x+1)$
40. $\ln\frac{(a+b)(a-b)}{c^2}$ or $\ln\left(\frac{a^2-b^2}{c^2}\right)$
42. $\log_5\left(\frac{xy^2}{z^3}\right)^2$ or $\log_5\left(\frac{x^2y^4}{z^6}\right)$
44. $\log_4 \frac{bd^c}{5^r}$

46. $\frac{\log 2}{\log 5} \approx .4307$
48. ≈ 2.5237
50. ≈ 3.5052

26. $\frac{\log 112}{\log 2} - \frac{\log 7}{\log 2}$ or $\log_2 \frac{112}{7}$ then change of base $\boxed{=4}$

28. $\boxed{=-\frac{1}{2}}$ 30. $\log_{12} 144 \boxed{=2}$

Answers to odd problems:

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1. $\log_2 x + \log_2(x-1)$ **3.** $23\log 7$

5. $\log_2 A + 2\log_2 B$ **7.** $\log_3 x + \frac{1}{2}\log_3 y$

9. $\frac{1}{3}\log_5(x^2+1)$ **11.** $\frac{1}{2}(\ln a + \ln b)$

13. $3\log x + 4\log y - 6\log z$

15. $\log_2 x + \log_2(x^2+1) - \frac{1}{2}\log_2(x^2-1)$

17. $\ln x + \frac{1}{2}(\ln y - \ln z)$ **19.** $\frac{1}{4}\log(x^2+y^2)$

21. $\frac{1}{2}[\log(x^2+4) - \log(x^2+1) - 2\log(x^3-7)]$

23. $\frac{1}{2}\ln x + 4\ln z - \frac{1}{3}\ln(y^2+6y+17)$

25. $\frac{3}{2}$ **27.** 1 **29.** 3 **31.** $\ln 8$ **33.** 16

35. $\log_3 160$ **37.** $\log_2(AB/C^2)$ **39.** $\log\left[\frac{x^4(x-1)^2}{\sqrt[3]{x^2+1}}\right]$

41. $\ln[5x^2(x^2+5)^3]$

43. $\log\left[\sqrt[3]{2x+1}\sqrt{(x-4)/(x^4-x^2-1)}\right]$

45. 2.807355 **47.** 2.182658 **49.** 0.655407

51. 4.165458